Synopsis For Ph.D. Registration

Faculty- Science

Subject - Mathematics

Title of the Project : "Strong S_1-Near-Rings, r-Regular Near-Rings, P-Regular Near-Rings and its Related Algebraic Structures."

Work Plan:

(a)Purpose and need of the study : The concept of regularity condition of a ring was introduced by von Neumann [1] in 1936. A ring is von Neumann regular if for every element x in the ring , there exists an element y such that x y x =x. The first step towards near-ring was an axiomatic work done by Dickson in 1905. After some time this ideas were shared by Zassenhaus with their studies of near-fields and by Wielandt with his important class of abstract near-rings. Various characterization of near-ring are developed by Betsch, Laxton, Ligh and others. The theory of near-rings has been developed much and at present it becomes a sophisticated theory with numerous applications in various areas like geometry, interpolation theory, group theory, polynomials and matrices. Designs are an important application of near-ring. The use of planar ring to get excellent balanced incomplete designs and experimental design is the best known application of near-rings to the outside world. Recently it is connected with computer science, automata, dynamical system, coding theory, cryptography. We shall study the concept of strongly s-near-rings, r-regular near-rings which are types of near-ring with specific algebraic properties. Recent research in near-rings and regular rings includes work on structure theory, generalized regularity concept and the application of graph theory to these algebraic structures, s-weakly regular and P-strongly regular near-rings. The behavior of these properties in near-rings, which lacks one of the distributive laws, is a central theme. In my present work it is proposed to generalize some specific results on strongly S_1-near-rings, r-regular near-rings and P-regular near-rings which has wide applications in the theory of abstract algebra and also in other branch of mathematics.

(b) Chapterwise Details:

Historically , the first step toward near-rings was an axiomatic research done by Dickson [2] in 1905. He showed that there do exist ''fields with only one distributive law'', known as near-field. After some years these near-fields showed

up again and proved to be useful in coordinatizing certain important classes of geometric planes said to the Descarte's method of coordinatizing the usual plane by the field of real numbers was one of the most successful steps in geometry. After some time Zassenhaus [3] was able to determine all finite near-fields so that near-fields are mightly tool in characterizing doubly transitive groups, incidence groups and Frobenius groups.

Chapter-I : Near- rings:

Algebraic system with addition and multiplication satisfying all the axioms of the skew-field except possibly one of the distributive laws have been studied occasionally. Dickson [2] studied and gave examples which showed that such system-called Fast Korper by Zassenhaus-an actually be non distributive. Zassenhaus [3] prove that Dickson's examples include all but seven of the finite Fastkorper which are not skew fields. All continuous Fastkorper of finite degree over the reals were found by Kalscheur [4]. Reidemister [5] connected Fastkorper with geometry of webs. All these results are in a certain sense extensions of well known facts about fields. In a related development, an algebraic system with two binary operations satisfying all the axioms of a ring except the commutative law of addition and one of the distributive laws were studied under the name 'Fastring' which was renamed as a 'Near-ring'. Moreover a near-ring satisfying the truncated distributive law $x(y+z) = x.y + x.z$, will be said to be a left near-ring to distinguish it from near-rings satisfying the right distributive law $(y+z).x = y.x + z.x$ called the right near-ring.

Chapter-II: Regular Near-Rings :

The concept of regular near-ring was first studied by Beidleman [6]and after some time by Heatherly [7]. After passes of time many related concepts of regular near-ring were studied by Goyal and Choudhary [8], Dheena [9] and others. In this section we shall introduce the concept of a regular near-ring with identity by imposing regularity condition over a near-ring and developed several generalized characterizations of a such rings. The ordinary ideas of regular near-

ring was first developed by Beidleman[7]. An element a of a near-ring R with identity is said to be regular if there exist an element x of R such that $axa = a$.

Chapter-III : Strongly S_1-near rings:

Strongly s-near-rings is likely referring to either strong S_1-near rings introduced by Silviya & Balakrishnan [10] or strongly regular near-rings, which are types of near-rings with specific algebraic properties. A near-ring N is an is an S_1-near-ring if for every a$\in N$, there exists an x$\in N^*$ (where $N^*=N \setminus \{0\}$) such that a x a = x a also a strong S_1-near-ring for every a $\in N$, the set { x $\in N^*$| a x a = x a }. We study some of its important properties and obtain a characterization and structure theorem under certain conditions. We find a condition for an S_1-near ring is regular. Every strong S_1-near ring is an S_1-near ring. A strong S_1- near ring is zero symmetric. Any homomorphic image of a strong S_1- near ring is a strong S_1-near ring. Every strong S_1- near ring is isomorphic to a subdirect product of sub directly irreducible strong S_1- near ring. We will prove that a strong S_1-near ring with mate function then it is sub directly irreducible if and only if it is simple.

Chapter-IV: r-regular near-rings :

The principal object ''m-regular near-ring' was cited by Krishna Murthy & Geetha and proved some results. We introduce 'm-regular near-ring with r-regular near-ring' and give some characterization. A r-regular near-ring property have a specific definition but a common proof involves demonstrating that every element can be represented by an idempotent element within its principal ideals. An r-regular near-ring is a type of near-ring where every element a is associated with a specific element e which satisfies the condition a = a e. This is different from a standard near-ring, where the condition is a = a x a . A near-ring N is called r-regular near-ring if for each a $\in N$ there exist $e^2 = e \in N$ such that a = ae . Any regular near-ring is a r-regular near-ring. If a near-ring is r-regular near-ring then every principal ideal is generated by an idempotent. If a near-ring is r-regular near-ring then it is symmetric and reduced. We also study ideal theory in r-regular near-rings.

Chapter-V: P-regular near-rings :

The concept of P-regular near-ring was introduced by Andrunakievich which is the generalization of regularity in rings. S.J.Choi extended the P-regularity in rings to P- regularity in near-rings, in which we find that a P-regular near-ring is

a near-ring where every for every element a, there exist an element x such that a= a x a + p for some element p in a specific ideal P of the near-ring. This is a generalization of a standard regular near-ring, which is a P-regular near-ring where the ideal P is the zero ideal (P=0). If P = {0}, the zero ideal then the definition of a regular near-ring becomes a regular near-ring, where a x a =a for some x∈ N. This concept can be extended to a related ideas like strongly P-regular near-rings and weakly P- regular near-rings, which have additional properties. So we find that every regular near-ring is P-regular for P = {0}.

(c) Hypothesis :

A hypothesis is an assumption, an idea which is proposed for the sake of argument so that it can be tested to see if it might be true. In the scientific method, the hypothesis is constructed before any applicable research has been done, apart from a basic background review. A simple hypothesis in a statement made to reflect the relation between exactly two variable which is independent and dependent. The present research work depend on the Zassenhaus [3] , Piltz [12], Beidleman [7] study on near-rings.

(d) Methodology :

Research methodology is the specific procedures or techniques used to identify, select, process and analyze information about a topic. The methodology also includes an explanation of data collection process. In my present work we shall study the books on Regular rings by K.R. Goodearl and Near-Ring' written by G.Piltz which contains several research papers concerning the topics of my work. Also we shall study the different books on abstract algebra like Herstein, Jacobson, Artin, Luthar and others and also different research paper published in the different national and international journals through internet and library related to the topic concerned

(e) References :

1. Von Neumann, J : 'On Regular Ring'
   Proc. Nat. Acad. Sc. USA Vol. 22,p. 707-713, 1936
2. Dickson L.E. : ''On finite algebras ''
   Nachr. Akad. Wiss. Gottingen, 358-393, 1905.
3. Zassenhaus H.: ''Uber endliche Fastkorper ''
   Abh. Math. Sem. Univ. Hamburg 11, 187-220, 1935/36

4. Kalscheur, F. :' Die Bestmmung..... Grundkorper' ' Abh. Math. Sem. Hamb. Univ. Vol.13, p. 413-415, 1940.'
5. Reidemuster : 'Grundlagen do Geometric' Berlin, p. 110-122, 1930.
6. Beidleman, J.C. :' A note on Regular Near-Ring' Jr. Indian Math. Soc. Vol. 33, p.207-210, 1969.
7. Heatherly, H.E. :' Regular Near-Rings.' Jr. Indian Math. Soc. Vol. 38, p. 345-354, 1974.
8. Goyal A.K. & Choudhary, S.C. : 'Generalized Regular Near-Rings'. Studia Scien. Math. Hung. Vol. 14, p. 69-76, 1979.
9. Dheena, P.: 'A generalization of Strongly Regular Near-Rings. Ind. Jr. Pure Applied Math. Vol. 20(1), p. 58-63, 1989
10. Silviya, Balskrishnan & Chelvam: Strong S_1-Near Rings International Jr. of Algebra, Vol 4, no.14, 685-691, 2010.
11. Reddy,Y.V.&Murty,C.V.L.N.:OnStronglyRegularNear-Rings. Proc. Edinb. Math. Soc. Vol. 27. p. 61-64, 1984.
12. Piltz, G. : 'Near-Rings', North Holland Amusterdam, 1983

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